INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2019-20
Statistics - IV, Midterm Examination, February 27, 2020
Marks are shown in square brackets. Total Marks: 50

1. Consider an $I \times J$ contingency table where the (i, j) cell has probability p_{ij} and observed count n_{ij}. Find the maximum likelihood estimate of p_{ij}
(a) when no restrictions are placed on the row and column factors;
(b) when it is known that the row and column factors are independent. [10]

2. Let $U_{(i)}^{(n)}$ denote the ith order statistics from a random sample of size n from $U(0, 1)$. Show that, for each i, $1 \leq i \leq n$,
$U_{(i)}^{(n)} - \frac{i}{n} \longrightarrow 0$ in probability as $n \longrightarrow \infty$. [5]

3. Consider a random sample $X_1, X_2, \ldots, X_n$ from a continuous distribution with c.d.f. F and suppose we want to test $H_0 : F = F_0$ where F_0 is a fully specified c.d.f. Define the directional and non-directional Kolmogorov-Smirnov test statistics, D_n^+, D_n^- and D_n for testing H_0. Show that, under H_0,
(a) D_n^- is distribution free;
(b) D_n^- converges to 0 in probability as $n \longrightarrow \infty$. [15]

4. Two methods, A and B, were used in a determination of the latent heat of fusion of ice. The investigators wished to check whether the methods differed, and if so, whether method B typically gave a higher reading. The following table gives the change in total heat from ice at $-.72°$C to $0°$C.

Method A	79.97	80.01	79.95	80.02	79.94
Method B	80.05	79.98	80.04	80.03	

Use an appropriate nonparametric method for this investigation. [10]

5. Suppose we have a random sample $X_1, \ldots, X_n$ from a continuous distribution with c.d.f. F and density f, both of which are completely unknown.
(a) Define the histogram estimate of f.
(b) Show that the histogram is a consistent estimator of f if the interval width is chosen to be proportional to $1/\log(n)$. [10]